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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2017

SECOND YEAR [BATCH 2016-19]

Date : 22/12/2017 MATHEMATICS FOR ECONOMICS [General]

Time : 11 am – 2 pm **Paper : III** Full Marks : 75

**[Use a separate Answer Book for each group]**

## Group – A

***Answer any four questions from Question nos. 1 to 6 :*** [4×5]

1. If $f(x,y)=\begin{cases}\dfrac{x^6-2y^4}{x^2+y^2}, & x^2+y^2\neq 0\\ 0 &, x^2+y^2=0\end{cases}$, show that $f(x, y)$ is differentiable at (0,0). 5

2. (a) Let $w=f(x,y,z)=yz-e^x$, where $x, y, z$ are the functions of $t$ given by,
   $x(t)=t^2,\ y(t)=t^2+1,\ z(t)=t$. Find $\dfrac{dw}{dt}$ at $t = 1$. 3
   
   (b) If the variables $P$, $V$ and $T$ are related by the equation $PV=nRT$, where $n$ and $R$ are constants, then what is the value of $\dfrac{\partial V}{\partial T}\cdot\dfrac{\partial T}{\partial P}\cdot\dfrac{\partial P}{\partial V}$. 2

3. (a) Define homogeneous function of degree $n$ (in case of three variables). 1
   
   (b) State and prove Euler theorem for homogeneous function for the function $f(x,y,z,t)$. 4

4. (a) State implicit functions theorem for a real valued continuous functions. 2
   
   (b) $f(x,y)=x^2+xy+y^2-1=0$ in the neighbourhood of $(0,-1)$. Find implicit function in term of $x$ i.e. $y=\phi(x)$. 3

5. Determine the maxima and minima of the function $f(x,y,z)=x^2-y^2$ on the surface $x^2+2y^2+3z^2=1$, by using Lagrange's method of undetermined multipliers. 5

6. (a) Show that $f(x,y)=x\sin\left(4\tan^{-1}\dfrac{y}{x}\right)$ for $x > 0$ and $f(0,y)=0$ for all $y$, is NOT differentiable at (0,0). 3
   
   (b) Prove that $f(x)=|x|$ has minimum at $x = 0$. 2

***Answer any two questions from Question nos. 7 to 9 :*** [2×10]

7. (a) Show that the curve $y=3x^5-40x^3+3x-20$ is concave upwards in $-1<x<0$ and $2<x<\infty$ but concave downwards in $-\infty<x<-2$ and $0<x<1$. Show also that $x=-2,0,2$ are points of inflection. 5
   
   (b) Prove that arbitrary union of closed subsets of $\mathbb{R}^2$ may not be closed. (Enough to give a counter example). 3
   
   (c) Is the set made up of all points of the $x$-axis together with all points of the $y$-axis closed? Justify your answer. 2

8. (a) Define directional derivative of $f(x,y)$ at $(a,b)$. Find the directional derivative of $f(x,y,z)=\sin(x+y^2)+z$ in the direction of the vector $(1,2,-1)$ when $(x,y,z)=(1,1,1)$. 5

(b) Give a set of sufficient conditions for the commutativity of the order of mixed derivatives at a point. 5

Let $f(x,y)=x^2\tan^{-1}\frac{y}{x}-y^2\tan^{-1}\frac{x}{y},\ xy\neq 0$

$$f(x,0)=0,\ f(0,y)=0\left[-\frac{\pi}{2}\leq\tan^{-1}\frac{x}{y},\ \tan^{-1}\frac{y}{x}\leq\frac{\pi}{2}\right]$$

Show that $f_{xy}(0,0)\neq f_{yx}(0,0)$.

9. (a) State fundamental theorem of Integral Calculus.

Evaluate $\lim_{x\to 0}\dfrac{\int_0^{x^2} e^{\sqrt{1+t}}\,dt}{x^2}$. 5

(b) Define 'Beta function' $\left(B(m,n);\ m>0,\ n>0\right)$ and 'Gamma function' $\left(\Gamma(n);\ n>0\right)$.

State the relation between Beta and Gamma function. Using this relation find $\Gamma\left(\frac{1}{2}\right)$ $\left(\text{Given } B\left(\frac{1}{2},\frac{1}{2}\right)=\pi\right)$, then prove that $\int_0^\infty e^{-x^2}dx=\frac{\sqrt{\pi}}{2}$. 5

## Group – B

***Answer any seven questions from Question Nos. 10 to 20*** : [7×5]

10. a) Find the order and degree of the differential equation

$$\frac{d^3y}{dx^3}+\left(\frac{dy}{dx}\right)^2=3\left\{1+\frac{d^2y}{dx^2}\right\}^{5/3}.$$ 2

b) Define an exact differential equation. Verify whether the following differential equation is exact or not?

$$\frac{dy}{dx}=\frac{e^{x^2}+ye^{x+y^2}}{e^{x^3+y^3}}.$$ 3

11. a) Define a first order linear differential equation and give an example. 2

b) Solve: $\frac{dy}{dx}-2xy=2x^3y^2$. 3

12. a) State the existence theorem on the solution of the ordinary differential equation $\frac{dy}{dx}=f(x,y)$. 2

b) Show that $\frac{1}{x^4}$ is an integrating factor of the differential equation: $\left(xy^2-e^{1/x^3}\right)dx-x^2y\,dy=0$. 3

13. a) Define a second order homogeneous linear differential equation and give an example. 2

b) Solve: $4\frac{d^2y}{dx^2}+12\frac{dy}{dx}+9y=0$. 3

14. Find the general and singular solution of the differential equation: $y=px+p-p^2$, where $p=\frac{dy}{dx}$. 2+3

15. Convert the following into a system of first order linear differential equation: 5

$$\frac{d^4y}{dx^4}+3\frac{d^3y}{dx^3}-\frac{\pi}{2}\frac{d^2y}{dx^2}+2\pi\frac{dy}{dx}-6y=1$$

16. Solve $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=0$ which satisfies the condition $y = 1$ and $\frac{dy}{dx}=0$ when $x = 0$. 5

17. Solve: $\frac{dy}{dx}+\frac{4x}{x^2+1}y=\frac{1}{\left(x^2+1\right)^3}$. 5

18. Solve: $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x$. 5

19. Solve, by the method of undetermined coefficients, the equation $\frac{d^2y}{dx^2}+y=10e^{2x}$, with the condition that $y = 0$ and $\frac{dy}{dx}=0$ when $x = 0$. 5

20. Apply the method of variation of parameter to solve the differential equation $\frac{d^2y}{dx^2}+4=\sin 2x$. 5

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